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Journal of Sound and Vibration 317 (2008) 687–708

JOURNAL OF SOUND AND VIBRATION

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qP-wave at a corrugated interface between two dissimilar pre-stressed elastic half-spaces

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Received 17 December 2006; received in revised form 29 August 2007; accepted 22 March 2008
Handling Editor: L.G. Tham
Available online 12 May 2008

Abstract

A plane qP-wave (quasi-P-wave) is assumed to be incident at a corrugated interface between two dissimilar pre-stressed elastic solid half-spaces. Using Rayleigh's method of approximation, the reflection and transmission coefficients have been presented for the first-order approximation of the corrugation. These coefficients are obtained in closed form for a corrugated interface of periodic shape. We found that these coefficients depend on the angle of incidence, frequency of the incident wave, initial stresses and incremental elastic properties of the half-spaces. The coefficients corresponding to irregularly reflected and transmitted waves are found to be proportional to the amplitude of the corrugated interface and are also influenced significantly by the initial stresses of the half-spaces. Some more results including the results of Sidhu and Singh [Reflection of P and SV-waves at the free surface of a prestressed elastic half-space, *Journal of the Acoustical Society of America* 76(2) (1984) 594–598] have been deduced as particular cases from the present problem.

1. Introduction

The theory of elastic wave propagation finds numerous applications in geophysics and seismology. They are of great help in exploration of the internal composition of the Earth layers and their properties. Seismic waves originating from the sources are to travel through different layers of the Earth and the velocities of these waves depend on the characteristics of the layer material through which they pass. While traveling through one layer to another adjacent layer, they undergo reflection and transmission at the interface between the layers. The reflection and transmission phenomenon of elastic waves not only depend on the layers' properties and angle of incidence but also depend on the shape of the interface. It is believed that the interface between any two adjacent layers of the Earth is not perfectly plane, but it is undulated in nature. Thus, while investigating the problems of reflection and transmission of elastic waves from such interfaces, the geometry of the interface should be taken into account.

A number of problems of reflection and transmission of elastic waves from plane boundaries have been investigated in the past, but a few problems have been attempted at the corrugated interface. For the first time,

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2008.03.036

Rayleigh [1] discussed a problem of reflection and transmission of waves from an undulated boundary surface. He obtained reflection and refraction coefficients of sound and light waves incident upon a corrugated boundary surface. In his method, the amplitude and slope of the corrugated interface are assumed to be small and the expression of the corrugated interface defining the boundary surface is expanded into Fourier series and the unknown coefficients in the boundary conditions are determined to the *n*th order of approximation in terms of small parameter characteristics of the corrugated interface. Later on, many researchers applied his method in various other fields to explain the reflection and transmission phenomena of waves at an irregular boundary surface. Some problems of reflection and refraction of elastic waves at a corrugated boundary surface have been studied using different techniques. Asano [2,3], Abubakar [4], Gupta [5], Lavy and Deresiewicz [6], Tomar and his coworkers [7–10] are some notable references.

Biot [11,12] gave the equations of wave motion and constitutive relations for a pre-stressed elastic medium and investigated the possibility of wave propagation. Since then many researchers have attempted a number of problems in the pre-stressed media. Babich [13] discussed the propagation of surface waves in a pre-stressed medium and showed that the velocity of surface waves varies linearly with the initial stress for a fixed frequency. Dowaikh and Ogden [14] studied the problem of interfacial (Stoneley) waves along the boundary between two half-spaces of pre-stressed incompressible isotropic elastic materials. They derived the secular equation and obtained a condition for the existence of a unique interfacial wave. Ogden and Sotiropoulos [15] discussed the problem of interfacial waves along the plane boundary between a pre-stressed incompressible elastic solid half-space and a pre-stressed incompressible elastic solid layer of uniform thickness. They derived the dispersion equation and obtained the conditions on the pre-strain, pre-stress and material parameters that ensure the existence of a unique interfacial wave speed at low and high frequencies. Khurana and Vashisth [16] analyzed a problem of Love wave propagation in a pre-stressed elastic layer overlying a pre-stressed poroelastic solid half-space. The interface between the layer and the half-space is considered to be loosely bonded, in general. They derived the frequency equation for Love wave propagation and studied the effects of the looseness of the interface and the initial stress on the phase velocity of Love waves. Pal and Chattopadhyay [17] discussed a problem of reflection of plane harmonic waves from a free boundary of a homogeneous, pre-stressed, orthotropic elastic half-space. It is shown that under certain conditions an incident pure mode of a P or SV-wave in a specific direction does not give rise to a pure mode of a reflected P-wave. Either there is reflection of superposition of P and SV-waves or an SV-wave only. Chattopadhyay et al. [18] discussed wave propagation in a pre-stressed elastic medium and studied a problem of reflection of P and SV waves at a free surface of an initially stressed elastic half-space. Norris [19] pointed out that Chattopadhyay et al. [18] assumed a form of solution which does not satisfy their equations of motion and the results obtained by them are in doubt. Norris [19] re-investigated the propagation of plane waves in a homogeneous pre-stressed elastic medium having an initial axial stress in two orthogonal directions. He showed that pure longitudinal and shear waves can propagate only in certain specific directions, which are defined. Sidhu and Singh [20] also commented on the paper by Chattopadhyay et al. [18] and stated that their results are not acceptable because the method of potentials used in the paper is not acceptable for pre-stressed media. Later, Sidhu and Singh [21] investigated the propagation of plane waves in a pre-stressed elastic solid with incremental elastic coefficients possessing orthotropic symmetry. They showed that two types of plane waves called a quasi-P wave and a quasi-S wave can exist and their velocities depend on the angle of propagation. They have also investigated the problem of reflection of P and SV-waves at the free surface of a pre-stressed elastic half-space. Subsequently, Sidhu and Singh [22] obtained a condition on the incremental elastic constants for the existence of real values of the phase velocity of quasi-S plane waves in a pre-stressed elastic solid. It is also shown that if this condition is violated, the quasi-S waves do not exist for a certain range of the angle of propagation. The propagation of elastic waves in an infinite pre-stressed elastic solid medium has also been investigated by Dahlen [23] and Tolstoy [24].

In this paper, we have attempted a problem of reflection and transmission of a quasi-P wave (called qP-wave) incident obliquely at a corrugated interface between two dissimilar pre-stressed elastic half-spaces. The reflection and transmission coefficients are obtained using Rayleigh's method of approximation by assuming that the amplitude and slope of the corrugated interface are small. The closed-form formulae of these coefficients for the first-order approximation are then presented for a particular type of interface (cosine law interface). Some more results including the results of Sidhu and Singh [21] have been deduced as particular cases from the present problem.

2. Problem formulation and equations

Consider the Cartesian x- and z-axes perpendicular to each other and lying on the horizontal plane, while the y-axis is vertical to this plane with its positive direction pointing downward. Suppose the two homogeneous pre-stressed half-spaces, namely M' and M' occupy the regions $-\infty < y \leqslant \zeta(x)$ and $\zeta(x) \leqslant y < \infty$, respectively, and are separated by a corrugated interface, $y = \zeta(x)$. The Fourier series expansion of this corrugated surface is given by

$$\zeta(x) = \sum_{n=1}^{\infty} (\zeta_n e^{\imath npx} + \zeta_{-n} e^{-\imath npx}), \tag{1}$$

where $\zeta(x)$ is a periodic function of x and independent of z, whose mean value is zero, ζ_n and ζ_{-n} are Fourier expansion coefficients, n is the series expansion order, $\imath = \sqrt{-1}$ and the wavelength of corrugation is given by $2\pi/p$.

We shall denote the parameters in the half-space M without a prime and those in the half-space M' with a prime. These half-spaces are either isotropic in finite strain or anisotropic with an orthotropic symmetry. By isotropic in finite strain, we mean that the stress is related to the finite strain by the relations which are independent of the orientation of the stress field. From the viewpoint of the incremental stresses the medium is isotropic in the vicinity of the unstressed state, but it will become anisotropic in a state of finite strain considered as the initial state. The coordinate systems are chosen to coincide with the principal directions of the initial stress which is then represented by its three principal components. Since the medium is isotropic in finite strain, the principal directions of stress define three planes of symmetry for the incremental elastic properties. This means that the incremental stress–strain relations must possess orthotropic symmetry (see Biot [14], p. 89). The state of initial stress is therefore defined by the principal components S_{11}, S_{22} and S_{33} in the half-space M and by S'_{11}, S'_{22} and S'_{33} in the half-space M'. If we restrict our analysis only to plane strain parallel to the xy-plane with the displacement components U and U' in the x direction and V and V' in the y direction then the third principal stresses S_{33} and S'_{33} do not enter explicitly in the equations of motion. Following Biot [12], the equations of motion for qP-wave propagation in the medium M are given by

$$B_{11}\frac{\partial^2 U}{\partial x^2} + A_3\frac{\partial^2 V}{\partial x\,\partial y} + A_1\frac{\partial^2 U}{\partial y^2} = \rho\frac{\partial^2 U}{\partial t^2}, \quad B_{22}\frac{\partial^2 V}{\partial y^2} + A_3\frac{\partial^2 U}{\partial x\,\partial y} + A_2\frac{\partial^2 V}{\partial x^2} = \rho\frac{\partial^2 V}{\partial t^2}, \tag{2}$$

where $A_1 = Q + P/2,\ A_2 = Q - P/2,\ A_3 = B_{12} + A_2,\ B_{12} = B_{21} + P,\ P = S_{22} - S_{11},\ \rho$ is the density of the medium M and B_{11}, B_{22}, B_{12} and Q are the incremental elastic coefficients.

Similarly, adopting the corresponding notations in the medium M', the equations of motion for qP-waves are given by

$$B'_{11}\frac{\partial^2 U'}{\partial x^2} + A'_3\frac{\partial^2 V'}{\partial x\,\partial y} + A'_1\frac{\partial^2 U'}{\partial y^2} = \rho'\frac{\partial^2 U'}{\partial t^2}, \quad B'_{22}\frac{\partial^2 V'}{\partial y^2} + A'_3\frac{\partial^2 U'}{\partial x\,\partial y} + A'_2\frac{\partial^2 V'}{\partial x^2} = \rho'\frac{\partial^2 V'}{\partial t^2}. \tag{3}$$

3. Reflection and transmission

Let a plane elastic wave propagating through M with phase velocity c be incident at the corrugated interface making an angle θ_0 with the normal. Sidhu and Singh [21] have shown that there exist two types of plane waves propagating with velocities given by

$$2\rho c_{1,2}^2 = E_1(\theta_0) + E_2(\theta_0) \pm \sqrt{(E_1(\theta_0) - E_2(\theta_0))^2 + 4A_3^2 \sin^2\theta_0 \cos^2\theta_0}, \tag{4}$$

where $E_1(\theta_0) = B_{11}\sin^2\theta_0 + A_1\cos^2\theta_0$ and $E_2(\theta_0) = B_{22}\cos^2\theta_0 + A_2\sin^2\theta_0$.

Clearly, the velocities given by c_1^2 (with upper sign) and c_2^2 (with lower sign) depend on the angle θ_0. Out of these two velocities, the larger one given by $c_1(\theta_0)$ represents the velocity for qP-waves and the smaller one given by $c_2(\theta_0)$ represents the velocity for qSV-waves in the pre-stressed elastic medium.

When a plane qP-wave becomes incident at the corrugated interface, there are irregularly reflected and transmitted waves due to corrugation of the interface, in addition to the regularly reflected and transmitted qP

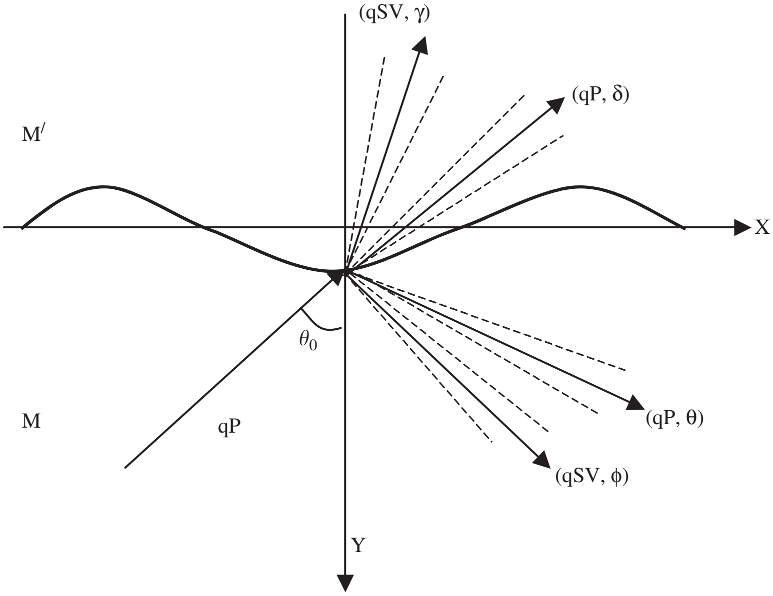

Fig. 1. Geometry of the problem.

and qSV-waves (see Asano [2]). These regularly reflected and transmitted waves are the same waves, which arise due to plane interface. The irregularly reflected and transmitted waves are those scattered waves which propagate with the same velocity as regular waves and appear on the left and right sides of the regular waves. The geometry of the problem is given in Fig. 1.

Thus, the total displacement in the medium M is given by the sum of the displacements caused by the incident wave, the regularly reflected waves and the irregularly reflected waves as follows:

$$U = G_0 e^{\iota J_0} + G_1 e^{\iota J_1} + G_2 e^{\iota J_2} + \sum_{n=1}^{\infty}[G_{1n}^{+} e^{\iota J_{1n}^{+}} + G_{1n}^{-} e^{\iota J_{1n}^{-}} + G_{2n}^{+} e^{\iota J_{2n}^{+}} + G_{2n}^{-} e^{\iota J_{2n}^{-}}],$$

$$V = H_0 e^{\iota J_0} + H_1 e^{\iota J_1} + H_2 e^{\iota J_2} + \sum_{n=1}^{\infty}[H_{1n}^{+} e^{\iota J_{1n}^{+}} + H_{1n}^{-} e^{\iota J_{1n}^{-}} + H_{2n}^{+} e^{\iota J_{2n}^{+}} + H_{2n}^{-} e^{\iota J_{2n}^{-}}], \tag{5}$$

where G_0 and H_0 are the amplitude constants of horizontal and vertical components of the displacement due to incident qP-wave, $G_1, H_1, G_2, H_2, G_{1n}^{\pm}, H_{1n}^{\pm}, G_{2n}^{\pm}$ and $H_{2n}^{\pm}$ are the amplitude constants and $J_0, J_1, J_2, J_{1n}^{\pm}$ and $J_{2n}^{\pm}$ are, respectively, the phase factors of the incident qP-wave at an angle θ_0, regularly reflected qP-wave at an angle θ, regularly reflected qSV-wave at an angle ϕ, irregularly reflected qP-waves at angles $\theta_n^{\pm}$, the irregularly reflected qSV-waves at angles $\phi_n^{\pm}$ and are given by $J_0 = k\{c_1 t - (x\sin\theta_0 - y\cos\theta_0)\}, J_1 = k\{c_1 t - (x\sin\theta + y\cos\theta)\}, J_2 = k\{c_2 t - (x\sin\phi + y\cos\phi)\}, J_{1n}^{\pm} = k\{c_1 t - (x\sin\theta_n^{\pm} + y\cos\theta_n^{\pm})\}$ and $J_{2n}^{\pm} = k\{c_2 t - (x\sin\phi_n^{\pm} + y\cos\phi_n^{\pm})\}$, k is the wave number.

The relations between various amplitude constants of horizontal and vertical components of displacements are given by (see Sidhu and Singh [21])

$$G_0 = F_0 H_0, \quad G_1 = -F_1 H_1, \quad G_2 = -F_2 H_2, \quad G_{1n}^{\pm} = -F_{1n}^{\pm} H_{1n}^{\pm}, \quad G_{2n}^{\pm} = -F_{2n}^{\pm} H_{2n}^{\pm}, \tag{6}$$

where

$$F_0 = F_1 = \frac{A_3 \sin\theta\cos\theta}{E_1(\theta) - \rho c_1^2}, \quad F_{1n}^{\pm} = \frac{A_3 \sin\theta_n^{\pm}\cos\theta_n^{\pm}}{E_1(\theta_n^{\pm}) - \rho c_1^2}, \quad F_2 = \frac{A_3 \sin\phi\cos\phi}{E_1(\phi) - \rho c_2^2}, \quad F_{2n}^{\pm} = \frac{A_3 \sin\phi_n^{\pm}\cos\phi_n^{\pm}}{E_1(\phi_n^{\pm}) - \rho c_2^2},$$

$$E_1(\theta) = B_{11}\sin^2\theta + A_1\cos^2\theta, \quad E_1(\phi) = B_{11}\sin^2\phi + A_1\cos^2\phi, \quad E_1(\theta_n^{\pm}) = B_{11}\sin^2\theta_n^{\pm} + A_1\cos^2\theta_n^{\pm},$$

$$E_1(\phi_n^{\pm}) = B_{11}\sin^2\phi_n^{\pm} + A_1\cos^2\phi_n^{\pm}.$$

$$E_2(\theta) = B_{22}\cos^2\theta + A_2\sin^2\theta, \quad E_2(\phi) = B_{22}\cos^2\phi + A_2\sin^2\phi,$$

$$2\rho c_1^2(\theta) = E_1(\theta) + E_2(\theta) + \sqrt{\{E_1(\theta) - E_2(\theta)\}^2 + 4A_3^2\sin^2\theta\cos^2\theta},$$

$$2\rho c_2^2(\phi) = E_1(\phi) + E_2(\phi) - \sqrt{\{E_1(\phi) - E_2(\phi)\}^2 + 4A_3^2\sin^2\phi\cos^2\phi},$$

Similarly, the total displacement in the medium M' is given by the sum of displacements caused by regularly and irregularly transmitted waves as follows:

$$U' = G_3 e^{\iota J_3} + G_4 e^{\iota J_4} + \sum_{n=1}^{\infty}[G_{3n}^{+}e^{\iota J_{3n}^{+}} + G_{3n}^{-}e^{\iota J_{3n}^{-}} + G_{4n}^{+}e^{\iota J_{4n}^{+}} + G_{4n}^{-}e^{\iota J_{4n}^{-}}],$$

$$V' = H_3 e^{\iota J_3} + H_4 e^{\iota J_4} + \sum_{n=1}^{\infty}[H_{3n}^{+}e^{\iota J_{3n}^{+}} + H_{3n}^{-}e^{\iota H_{3n}^{-}} + H_{4n}^{+}e^{\iota J_{4n}^{+}} + H_{4n}^{-}e^{\iota J_{4n}^{-}}], \tag{7}$$

where $G_3, H_3, G_4, H_4, G_{3n}^{\pm}, H_{3n}^{\pm}, G_{4n}^{\pm}$ and $H_{4n}^{\pm}$ are the amplitude constants and $J_3, J_4, J_{3n}^{\pm}$ and $J_{4n}^{\pm}$ are, respectively, the phase factors of the regularly transmitted qP-wave at an angle δ, the regularly transmitted qSV-wave at an angle γ, the irregularly transmitted qP-waves at angles $\delta_n^{\pm}$, the irregularly transmitted qSV-waves at angles $\gamma_n^{\pm}$ and are given by $J_3 = k\{c_1't - (x\sin\delta - y\cos\delta)\}, J_{3n}^{\pm} = k\{c_1't - (x\sin\delta_n^{\pm} - y\cos\delta_n^{\pm})\}$, $J_4 = k\{c_2't - (x\sin\gamma - y\cos\gamma)\}, J_{4n}^{\pm} = k\{c_2't - (x\sin\gamma_n^{\pm} - y\cos\gamma_n^{\pm})\}$. Similar to the relations in (6), we have

$$G_3 = F_3H_3, \quad G_4 = F_4H_4, \quad G_{3n}^{\pm} = F_{3n}^{\pm}H_{3n}^{\pm}, \quad G_{4n}^{\pm} = F_{4n}^{\pm}H_{4n}^{\pm}, \tag{8}$$

where

$$F_3 = \frac{A_3'\sin\delta\cos\delta}{E_1'(\delta) - \rho'c_1'^2}, \quad F_{3n}^{\pm} = \frac{A_3'\sin\delta_n^{\pm}\cos\delta_n^{\pm}}{E_1'(\delta_n^{\pm}) - \rho'c_1'^2}, \quad F_4 = \frac{A_3'\sin\gamma\cos\gamma}{E_1'(\gamma) - \rho'c_2'^2}, \quad F_{4n}^{\pm} = \frac{A_3'\sin\gamma_n^{\pm}\cos\gamma_n^{\pm}}{E_1'(\gamma_n^{\pm}) - \rho'c_2'^2},$$

$$E_1'(\delta) = B_{11}'\sin^2\delta + A_1'\cos^2\delta, \quad E_1'(\gamma) = B_{11}'\sin^2\gamma + A_1'\cos^2\gamma, \quad E_1'(\delta_n^{\pm}) = B_{11}'\sin^2\delta_n^{\pm} + A_1'\cos^2\delta_n^{\pm},$$

$$E_1'(\gamma_n^{\pm}) = B_{11}'\sin^2\gamma_n^{\pm} + A_1'\cos^2\gamma_n^{\pm}, \quad E_2'(\delta) = B_{22}'\cos^2\delta + A_2'\sin^2\delta, \quad E_2'(\gamma) = B_{22}'\cos^2\gamma + A_2'\sin^2\gamma,$$

$$2\rho'c_1'^2 = E_1'(\delta) + E_2'(\delta) + \sqrt{\{E_1'(\delta) - E_2'(\delta)\}^2 + 4A_3'^2\sin^2\delta\cos^2\delta},$$

$$2\rho'c_2'^2 = E_1'(\gamma) + E_2'(\gamma) - \sqrt{\{E_1'(\gamma) - E_2'(\gamma)\}^2 + 4A_3'^2\sin^2\gamma\cos^2\gamma}.$$

Snell's law, which gives the relation of the angle of incidence with the angles of regularly reflected waves and the transmitted waves, is given by

$$\frac{\sin\theta_0}{c_1(\theta_0)} = \frac{\sin\theta}{c_1(\theta)} = \frac{\sin\phi}{c_2(\phi)} = \frac{\sin\delta}{c_1'(\delta)} = \frac{\sin\gamma}{c_2'(\gamma)} = \frac{1}{c_a}, \tag{9}$$

where c_a is the apparent velocity. Moreover, each angle of regularly reflected and transmitted waves is related to the angle of irregularly reflected and transmitted waves through the following Spectrum theorem (see Asano [3]) as

$$\sin\begin{Bmatrix}\theta_n^{\pm}\\ \phi_n^{\pm}\\ \delta_n^{\pm}\\ \gamma_n^{\pm}\end{Bmatrix} - \sin\begin{Bmatrix}\theta\\ \phi\\ \delta\\ \gamma\end{Bmatrix} = \pm\frac{np}{\omega}\begin{Bmatrix}c_1\\ c_2\\ c_1'\\ c_2'\end{Bmatrix}. \tag{10}$$

where ω is the angular frequency.

4. Boundary conditions

The appropriate boundary conditions are the continuity of displacement components and traction at the corrugated interface $y=\zeta(x)$. Mathematically, these boundary conditions can be expressed as: At $y=\zeta(x)$

$$V=V', \tag{11}$$

$$U=U', \tag{12}$$

$$-\zeta'\left(B_{11}\frac{\partial U}{\partial x}+B_1\frac{\partial V}{\partial y}\right)+A_{01}\frac{\partial V}{\partial x}+A_1\frac{\partial U}{\partial y}=-\zeta'\left(B'_{11}\frac{\partial U'}{\partial x}+B'_1\frac{\partial V'}{\partial y}\right)+A'_{01}\frac{\partial V'}{\partial x}+A'_1\frac{\partial U'}{\partial y} \tag{13}$$

and

$$-\zeta'\left(A_2\frac{\partial V}{\partial x}+A_{01}\frac{\partial U}{\partial y}\right)+B_1\frac{\partial U}{\partial x}+B_{22}\frac{\partial V}{\partial y}=-\zeta'\left(A'_2\frac{\partial V'}{\partial x}+A'_{01}\frac{\partial U'}{\partial y}\right)+B'_1\frac{\partial U'}{\partial x}+B'_{22}\frac{\partial V'}{\partial y}, \tag{14}$$

where $A_{01}=Q-(S_{22}+S_{11})/2, A'_{01}=Q'-(S'_{22}+S'_{11})/2, B_1=B_{12}+S_{11}$ and $B'_1=B'_{12}+S'_{11}$.

Inserting the values of U,V,U' and V' from Eqs. (5) and (7) into Eqs. (11)–(14), we obtain

$$\begin{aligned}&H_0e^{\iota\zeta R_0}+H_1e^{-\iota\zeta R}+H_2e^{-\iota\zeta Q}+\sum_{n=1}^{\infty}[\{H^+_{1n}e^{-\iota\zeta R^+_n}+H^+_{2n}e^{-\iota\zeta Q^+_n}\}e^{-\iota npx}+\{H^-_{1n}e^{-\iota\zeta R^-_n}+H^-_{2n}e^{-\iota\zeta Q^-_n}\}e^{\iota npx}]\\&=H_3e^{\iota\zeta S}+H_4e^{\iota\zeta L}+\sum_{n=1}^{\infty}[\{H^+_{3n}e^{\iota\zeta S^+_n}+H^+_{4n}e^{\iota\zeta L^+_n}\}e^{-\iota npx}+\{H^-_{3n}e^{\iota\zeta S^-_n}+H^-_{4n}e^{\iota\zeta L^-_n}\}e^{\iota npx}],\end{aligned} \tag{15}$$

$$\begin{aligned}&G_0e^{\iota\zeta R_0}+G_1e^{-\iota\zeta R}+G_2e^{-\iota\zeta Q}+\sum_{n=1}^{\infty}[\{G^+_{1n}e^{-\iota\zeta R^+_n}+G^+_{2n}e^{-\iota\zeta Q^+_n}\}e^{-\iota npx}+\{G^-_{1n}e^{-\iota\zeta R^-_n}+G^-_{2n}e^{-\iota\zeta Q^-_n}\}e^{\iota npx}]\\&=G_3e^{\iota\zeta S}+G_4e^{\iota\zeta L}+\sum_{n=1}^{\infty}[\{G^+_{3n}e^{\iota\zeta S^+_n}+G^+_{4n}e^{\iota\zeta L^+_n}\}e^{-\iota npx}+\{G^-_{3n}e^{\iota\zeta S^-_n}+G^-_{4n}e^{\iota\zeta L^-_n}\}e^{\iota npx}],\end{aligned} \tag{16}$$

$$\begin{aligned}&\{(\zeta'G_0B_{11}-A_{01}H_0)P_0+(-\zeta'B_1H_0+A_1G_0)R_0\}e^{\iota\zeta R_0}+\{(\zeta'G_1B_{11}-A_{01}H_1)P_0+(\zeta'B_1H_1\\&-A_1G_1)R\}e^{-\iota\zeta R}+\{(\zeta'G_2B_{11}-A_{01}H_2)P_0+(\zeta'B_1H_2-A_1G_2)Q\}e^{-\iota\zeta Q}+\sum_{n=1}^{\infty}[\{(P_0+np)(\zeta'G^+_{1n}B_{11}\\&-A_{01}H^+_{1n})+(\zeta'B_1H^+_{1n}-A_1G^+_{1n})R^+_n\}e^{-\iota\zeta R^+_n}e^{-\iota npx}+\{(P_0-np)(\zeta'G^-_{1n}B_{11}-A_{01}H^-_{1n})+(\zeta'B_1H^-_{1n}\\&-A_1G^-_{1n})R^-_n\}e^{-\iota\zeta R^-_n}e^{\iota npx}+\{(P_0+np)(\zeta'G^+_{2n}B_{11}-A_{01}H^+_{2n})+(\zeta'B_1H^+_{2n}-A_1G^+_{2n})Q^+_n\}e^{-\iota\zeta Q^+_n}e^{-\iota npx}\\&+\{(P_0-np)(\zeta'G^-_{2n}B_{11}-A_{01}H^-_{2n})+(\zeta'B_1H^-_{2n}-A_1G^-_{2n})Q^-_n\}e^{-\iota\zeta Q^-_n}e^{\iota npx}]=\{(\zeta'B'_{11}G_3-A'_{01}H_3)P_0\\&-(\zeta'B'_1H_3-A'_1G_3)S\}e^{\iota\zeta S}+\{(\zeta'B'_{11}G_4-A'_{01}H_4)P_0-(\zeta'B'_1H_4-A'_1G_4)L\}e^{\iota\zeta L}+\sum_{n=1}^{\infty}[\{(P_0\\&+np)(G^+_{3n}B'_{11}\zeta'-A'_{01}H^+_{3n})-(\zeta'B'_1H^+_{3n}-A'_1G^+_{3n})S^+_n\}e^{\iota\zeta S^+_n}e^{-\iota npx}+\{(P_0-np)(G^-_{3n}B'_{11}\zeta'\\&-A'_{01}H^-_{3n})-(\zeta'B'_1H^-_{3n}-A'_1G^-_{3n})S^-_n\}e^{\iota\zeta S^-_n}e^{\iota npx}+\{(P_0+np)(G^+_{4n}B'_{11}\zeta'-A'_{01}H^+_{4n})-(\zeta'B'_1H^+_{4n}-A'_1G^+_{4n})\\&\times L^+_n\}e^{\iota\zeta L^+_n}e^{-\iota npx}+\{(P_0-np)(G^-_{4n}B'_{11}\zeta'-A'_{01}H^-_{4n})-(\zeta'B'_1H^-_{4n}-A'_1G^-_{4n})L^-_n\}e^{\iota\zeta L^-_n}e^{\iota npx}],\end{aligned} \tag{17}$$

$$\begin{aligned}&\{(\zeta'H_0A_2-B_1G_0)P_0-(\zeta'A_{01}G_0-B_{22}H_0)R_0\}e^{\iota\zeta R_0}+\{(\zeta'H_1A_2-B_1G_1)P_0+(\zeta'A_{01}G_1\\&-B_{22}H_1)R\}e^{-\iota\zeta R}+\{(\zeta'H_2A_2-B_1G_2)P_0+(A_{01}G_2\zeta'-B_{22}H_2)Q\}e^{-\iota\zeta Q}+\sum_{n=1}^{\infty}[\{(P_0+np)\\&\times(\zeta'A_2H^+_{1n}-B_1G^+_{1n})+(\zeta'A_{01}G^+_{1n}-B_{22}H^+_{1n})R^+_n\}e^{-\iota\zeta R^+_n}e^{-\iota npx}+\{(P_0-np)(\zeta'A_2H^-_{1n}-B_1G^-_{1n})\end{aligned}$$

$$
\begin{aligned}
&+ (A_{01}G_{1n}^{-}\zeta' - B_{22}H_{1n}^{-})R_n^{-}\}\mathrm{e}^{-\imath\zeta R_n^{-}}\mathrm{e}^{\imath npx} + \{(P_0+np)(\zeta' A_2 H_{2n}^{+} - B_1 G_{2n}^{+}) + (A_{01}G_{2n}^{+}\zeta' - B_{22}H_{2n}^{+})Q_n^{+}\} \\
&\times \mathrm{e}^{-\imath\zeta Q_n^{+}}\mathrm{e}^{-\imath npx} + \{(P_0-np)(\zeta' A_2 H_{2n}^{-} - B_1 G_{2n}^{-}) + (\zeta' A_{01}G_{2n}^{-} - B_{22}H_{2n}^{-})Q_n^{-}\}\mathrm{e}^{-\imath\zeta Q_n^{-}}\mathrm{e}^{\imath npx}] \\
= &\{(\zeta' A_2' H_3 - B_1' G_3)P_0 - (\zeta' A_{01}' G_3 - B_{22}' H_3)S\}\mathrm{e}^{\imath\zeta S} + \{(\zeta' A_2' H_4 - B_1' G_4)P_0 - (\zeta' A_{01}' G_4 \\
&- B_{22}' H_4)L\}\mathrm{e}^{\imath\zeta L} + \sum_{n=1}^{\infty}[\{(P_0+np)(H_{3n}^{+}A_2'\zeta' - B_1' G_{3n}^{+}) - (\zeta' A_{01}' G_{3n}^{+} - B_{22}' H_{3n}^{+})S_n^{+}\}\mathrm{e}^{\imath\zeta S_n^{+}}\mathrm{e}^{-\imath npx} \\
&+ \{(P_0-np)(H_{3n}^{-}A_2'\zeta' - B_1' G_{3n}^{-}) - (\zeta' A_{01}' G_{3n}^{-} - B_{22}' H_{3n}^{-})S_n^{-}\}\mathrm{e}^{\imath\zeta S_n^{-}}\mathrm{e}^{\imath npx} + \{(P_0+np)(H_{4n}^{+}A_2'\zeta' \\
&- B_1' G_{4n}^{+}) - (\zeta' A_{01}' G_{4n}^{+} - B_{22}' H_{4n}^{+})L_n^{+}\}\mathrm{e}^{\imath\zeta L_n^{+}}\mathrm{e}^{-\imath npx} + \{(P_0-np)(H_{4n}^{-}A_2'\zeta' - B_1' G_{4n}^{-}) - (\zeta' A_{01}' G_{4n}^{-} \\
&- B_{22}' H_{4n}^{-})L_n^{-}\}\mathrm{e}^{\imath\zeta L_n^{-}}\mathrm{e}^{\imath npx}], \qquad (18)
\end{aligned}
$$

where

$$\zeta' = \sum_{n=1}^{\infty}(\zeta_n \mathrm{e}^{inpx} - \zeta_{-n}\mathrm{e}^{-inpx})\imath np,$$

$$P_0 = \frac{\omega\sin\theta_0}{c_1}, \quad R_0 = R = \frac{\omega\cos\theta}{c_1}, \quad R_n^{\pm} = \frac{\omega\cos\theta_n^{\pm}}{c_1}, \quad Q = \frac{\omega\cos\phi}{c_2}, \quad Q_n^{\pm} = \frac{\omega\cos\phi_n^{\pm}}{c_2},$$

$$S = \frac{\omega\cos\delta}{c_1'}, \quad S_n^{\pm} = \frac{\omega\cos\delta_n^{\pm}}{c_1'}, \quad L = \frac{\omega\cos\gamma}{c_2'}, \quad L_n^{\pm} = \frac{\omega\cos\gamma_n^{\pm}}{c_2'}.$$

These Eqs. (15)–(18) are the equations satisfying the boundary conditions which consist of unknown amplitude constants. Knowing the expressions of amplitude constants from these equations, one can obtain the reflection and transmission coefficients to the nth order of approximation of the corrugated interface. Here, we shall obtain these coefficients only for the first-order approximation of the corrugation.

5. Solution of the first-order approximation

Assuming that the amplitude and slope of the corrugated interface are so small that the higher powers of ζ can be neglected, we can write

$$\exp(\pm\imath\zeta R) \approx 1 \pm \imath\zeta R. \qquad (19)$$

Using Eqs. (6), (8)–(10) and (19) in Eqs. (15)–(18) and comparing the term independent of x and ζ to both sides of the resulting equations, we obtain

$$\frac{H_1}{H_0} + \frac{H_2}{H_0} - \frac{H_3}{H_0} - \frac{H_4}{H_0} = -1, \qquad (20)$$

$$F_1\frac{H_1}{H_0} + F_2\frac{H_2}{H_0} + F_3\frac{H_3}{H_0} + F_4\frac{H_4}{H_0} = F_1, \qquad (21)$$

$$-a_1\frac{H_1}{H_0} + a_2\frac{H_2}{H_0} + a_3\frac{H_3}{H_0} + a_4\frac{H_4}{H_0} = a_1, \qquad (22)$$

$$b_1\frac{H_1}{H_0} + b_2\frac{H_2}{H_0} + b_3\frac{H_3}{H_0} + b_4\frac{H_4}{H_0} = b_1, \qquad (23)$$

where

$$a_1 = P_0 A_{01} - RF_1 A_1, \quad a_2 = -P_0 A_{01} + QF_2 A_1, \quad a_3 = P_0 A_{01}' - SF_3 A_1', \quad a_4 = P_0 A_{01}' - LF_4 A_1',$$

$$b_1 = F_1 P_0 B_1 - B_{22} R, \quad b_2 = F_2 P_0 B_1 - B_{22} Q, \quad b_3 = F_3 P_0 B_1' - B_{22}' S \quad \text{and} \quad b_4 = F_4 P_0 B_1' - B_{22}' L.$$

Next, comparing the coefficient of $e^{-\imath npx}$ to both sides, we obtain

$$\frac{H_{1n}^{+}}{H_0}+\frac{H_{2n}^{+}}{H_0}-\frac{H_{3n}^{+}}{H_0}-\frac{H_{4n}^{+}}{H_0}=a_{10}^{+}, \tag{24}$$

$$F_{1n}^{+}\frac{H_{1n}^{+}}{H_0}+F_{2n}^{+}\frac{H_{2n}^{+}}{H_0}+F_{3n}^{+}\frac{H_{3n}^{+}}{H_0}+F_{4n}^{+}\frac{H_{4n}^{+}}{H_0}=a_{20}^{+}, \tag{25}$$

$$a_{1n}^{+}\frac{H_{1n}^{+}}{H_0}+a_{2n}^{+}\frac{H_{2n}^{+}}{H_0}+a_{3n}^{+}\frac{H_{3n}^{+}}{H_0}+a_{4n}^{+}\frac{H_{4n}^{+}}{H_0}=a_{0n}^{+}, \tag{26}$$

$$b_{1n}^{+}\frac{H_{1n}^{+}}{H_0}+b_{2n}^{+}\frac{H_{2n}^{+}}{H_0}+b_{3n}^{+}\frac{H_{3n}^{+}}{H_0}+b_{4n}^{+}\frac{H_{4n}^{+}}{H_0}=b_{0n}^{+}, \tag{27}$$

where

$$a_{10}^{+}=\imath\zeta_{-n}\left[-R+R\frac{H_1}{H_0}+Q\frac{H_2}{H_0}+S\frac{H_3}{H_0}+L\frac{H_4}{H_0}\right],\quad a_{20}^{+}=\imath\zeta_{-n}\left[F_1R+F_1R\frac{H_1}{H_0}+F_2Q\frac{H_2}{H_0}-F_3S\frac{H_3}{H_0}-F_4L\frac{H_4}{H_0}\right],$$

$$a_{1n}^{+}=-A_{01}(P_0+np)+F_{1n}^{+}A_1R_n^{+},\quad a_{2n}^{+}=-A_{01}(P_0+np)+F_{2n}^{+}A_1Q_n^{+},\quad a_{3n}^{+}=A_{01}'(P_0+np)-F_{3n}^{+}A_1'S_n^{+},$$

$$\begin{aligned}a_{4n}^{+}&=A_{01}'(P_0+np)-F_{4n}^{+}A_1'L_n^{+},\quad a_{0n}^{+}=\imath\zeta_{-n}\Big[B_{11}F_1npP_0-B_1npR+A_{01}P_0R-A_1F_1R^2+\{-B_{11}F_1npP_0\\&+B_1npR-A_{01}P_0R+A_1F_1R^2\}\frac{H_1}{H_0}+\{-B_{11}npF_2P_0+B_1npQ-A_{01}P_0Q+A_1F_2Q^2\}\frac{H_2}{H_0}+\{-B_{11}'F_3npP_0\\&+B_1'npS-A_{01}'P_0S+A_1'F_3S^2\}\frac{H_3}{H_0}+\{-B_{11}'F_4npP_0+B_1'npL-A_{01}'LP_0+A_1'F_4L^2\}\frac{H_4}{H_0}\Big],\end{aligned}$$

$$b_{1n}^{+}=B_1F_{1n}^{+}(P_0+np)-B_{22}R_n^{+},\quad b_{2n}^{+}=B_1F_{2n}^{+}(P_0+np)-B_{22}Q_n^{+},\quad b_{3n}^{+}=F_{3n}^{+}B_1'(P_0+np)-B_{22}'S_n^{+},$$

$$\begin{aligned}b_{4n}^{+}&=F_{4n}^{+}(P_0+np)B_1'-B_{22}'L_n^{+},\quad b_{0n}^{+}=\imath\zeta_{-n}\Big[A_2npP_0-A_{01}F_1npR+B_1F_1RP_0-B_{22}R^2\\&+\{npA_2P_0-F_1npA_{01}R+B_1F_1RP_0-B_{22}R^2\}\frac{H_1}{H_0}+\{A_2npP_0-F_2npA_{01}Q+B_1F_2QP_0-B_{22}Q^2\}\frac{H_2}{H_0}\\&+\{-npA_2'P_0+npF_3A_{01}'S-B_1'F_3S+B_{22}'S^2\}\frac{H_3}{H_0}+\{-A_2'npP_0+npF_4A_{01}'L-B_1'F_4LP_0+B_{22}'L^2\}\frac{H_4}{H_0}\Big].\end{aligned}$$

Likewise, comparing the coefficient of $e^{\imath npx}$ to both sides, we obtain

$$\frac{H_{1n}^{-}}{H_0}+\frac{H_{2n}^{-}}{H_0}-\frac{H_{3n}^{-}}{H_0}-\frac{H_{4n}^{-}}{H_0}=a_{10}^{-}, \tag{28}$$

$$F_{1n}^{-}\frac{H_{1n}^{-}}{H_0}+F_{2n}^{-}\frac{H_{2n}^{-}}{H_0}+F_{3n}^{-}\frac{H_{3n}^{-}}{H_0}+F_{4n}^{-}\frac{H_{4n}^{-}}{H_0}=a_{20}^{-}, \tag{29}$$

$$a_{1n}^{-}\frac{H_{1n}^{-}}{H_0}+a_{2n}^{-}\frac{H_{2n}^{-}}{H_0}+a_{3n}^{-}\frac{H_{3n}^{-}}{H_0}+a_{4n}^{-}\frac{H_{4n}^{-}}{H_0}=a_{0n}^{-}, \tag{30}$$

$$b_{1n}^{-}\frac{H_{1n}^{-}}{H_0}+b_{2n}^{-}\frac{H_{2n}^{-}}{H_0}+b_{3n}^{-}\frac{H_{3n}^{-}}{H_0}+b_{4n}^{-}\frac{H_{4n}^{-}}{H_0}=b_{0n}^{-}, \tag{31}$$

where

$$a_{10}^{-}=\imath\zeta_{n}\left[-R+R\frac{H_1}{H_0}+Q\frac{H_2}{H_0}+S\frac{H_3}{H_0}+L\frac{H_4}{H_0}\right],\quad a_{20}^{-}=\imath\zeta_{n}\left[F_1R+F_1R\frac{H_1}{H_0}+F_2Q\frac{H_2}{H_0}-F_3S\frac{H_3}{H_0}-F_4L\frac{H_4}{H_0}\right],$$

$$a_{1n}^- = -A_{01}(P_0 - np) + F_{1n}^- A_1 R_{1n}^-, \quad a_{2n}^- = -A_{01}(P_0 - np) + F_{2n}^- A_1 Q_n^-, \quad a_{3n}^- = A'_{01}(P_0 - np) - F_{3n}^- A'_1 S_n^-,$$

$$\begin{aligned} a_{4n}^- &= A'_{01}(P_0 - np) - F_{4n}^- A'_1 L_n^-, \quad a_{0n}^- = \iota\zeta_n\Big[- npB_{11}F_1P_0 + npB_1R + A_{01}RP_0 - A_1F_0R^2 \\ &+ \{(npF_1B_{11} - A_{01}R)P_0 + A_1F_1R^2 - npB_1R\}\frac{H_1}{H_0} + \{(B_{11}npF_2 - A_{01}Q)P_0 - npB_1Q + F_2A_1Q^2\}\frac{H_2}{H_0} \\ &+ \{(npB'_{11}F_3 - A'_{01}S)P_0 - B'_1npS + F_3A'_1S^2\}\frac{H_3}{H_0} + \{(npF_4B'_{11} - A'_{01}L)P_0 - B'_{11}npL + F_4A'_1L^2\}\frac{H_4}{H_0}\Big], \end{aligned}$$

$$b_{1n}^- = B_1F_{1n}^-(P_0 - np) - B_{22}R_n^-, \quad b_{2n}^- = B_1F_{2n}^-(P_0 - np) - B_{22}Q_n^-, \quad b_{3n}^- = F_{3n}^-B'_1(P_0 - np) + B'_{22}S_n^-,$$

$$\begin{aligned} b_{4n}^- &= F_{4n}^-(P_0 - np)B'_1 + B'_{22}L_n^-, \quad b_{0n}^- = \iota\zeta_n\Big[(-npA_2 + F_1B_1R)P_0 + F_1npA_{01}R - B_{22}R^2 + \{(-npA_2 \\ &- B_1F_1R)P_0 + A_{01}F_1npR - B_{22}R^2\}\frac{H_1}{H_0} + \{(-npA_2 + B_1F_2Q)P_0 + F_2A_{01}npQ - B_{22}Q^2\}\frac{H_2}{H_0} \\ &+ \{(A'_2np - B'_1F_3S)P_0 - F_3A'_{01}npS + B'_{22}S^2\}\frac{H_3}{H_0} + \{(npA'_2 - B'_1F_4L)P_0 - F_4npA'_{01}L + B'_{22}L^2\}\frac{H_4}{H_0}\Big]. \end{aligned}$$

Eqs. (20)–(23) give the formulae for amplitude constants at the plane interface between two different pre-stressed elastic half-spaces, while Eqs. (24)–(31) give that at a corrugated interface for the first-order approximation of the corrugation.

Solving Eqs. (20)–(23), we get the ratios of amplitude constants for vertical displacement component of the reflected and transmitted waves at the plane interface as

$$\frac{H_1}{H_0} = \frac{\Delta_{H1}}{\Delta}, \quad \frac{H_2}{H_0} = \frac{\Delta_{H2}}{\Delta}, \quad \frac{H_3}{H_0} = \frac{\Delta_{H3}}{\Delta}, \quad \frac{H_4}{H_0} = \frac{\Delta_{H4}}{\Delta} \tag{32}$$

where

$$\Delta = \begin{vmatrix} 1 & 1 & -1 & -1 \\ F_1 & F_2 & F_3 & F_4 \\ -a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{vmatrix}$$

and the expressions of $\Delta_{H1}, \Delta_{H2}, \Delta_{H3}$ and Δ_{H4} can be written by replacing the first, second, third and fourth columns of the above determinant in Δ with the column matrix $[-1 \;\; F_1 \;\; a_1 \;\; b_1]^t$, respectively. Using Eqs. (6) and (8), we obtain the expression of the ratios of amplitude constants for horizontal displacement components of the reflected and transmitted waves at the plane interface as

$$\frac{G_1}{G_0} = -\frac{\Delta_{H1}}{\Delta}, \quad \frac{G_2}{G_0} = -\frac{F_2}{F_1}\frac{\Delta_{H2}}{\Delta}, \quad \frac{G_3}{G_0} = \frac{F_3}{F_1}\frac{\Delta_{H3}}{\Delta}, \quad \frac{G_4}{G_0} = \frac{F_4}{F_1}\frac{\Delta_{H4}}{\Delta}. \tag{33}$$

The amplitude of the incident qP-wave is given by $\sqrt{H_0^2 + G_0^2} = \sqrt{1 + F_1^2}H_0$.

Therefore, the reflection coefficients R_{pp} (corresponding to the reflected qP-wave), R_{ps} (corresponding to the reflected qSV-wave), transmission coefficients T_{pp} (corresponding to the transmitted qP-wave) and T_{ps} (corresponding to the transmitted qSV-wave) are then given by

$$R_{\mathrm{pp}} = \frac{\Delta_{H1}}{\Delta}, \quad R_{\mathrm{ps}} = \sqrt{\frac{1 + F_2^2}{1 + F_1^2}}\frac{\Delta_{H2}}{\Delta}, \quad T_{\mathrm{pp}} = \sqrt{\frac{1 + F_3^2}{1 + F_1^2}}\frac{\Delta_{H3}}{\Delta}, \quad T_{\mathrm{ps}} = \sqrt{\frac{1 + F_4^2}{1 + F_1^2}}\frac{\Delta_{H4}}{\Delta}. \tag{34}$$

It is clear that these coefficients at the plane interface are independent of the amplitude of corrugation and the frequency of the incident wave.

After obtaining the amplitude constants corresponding to irregularly reflected and transmitted waves for the first-order approximation of the corrugated interface from Eqs. (24)–(31), the concerned expression of reflection and transmission coefficients can be calculated. We shall obtain these coefficients in the next section for a periodic type of interface.

6. A periodic interface

Let us take a simple periodic interface represented by only one cosine term, i.e., $\zeta = d\cos px$, where d is the amplitude of the corrugated interface and $2\pi/p$ is the wavelength of the corrugation. Comparing this equation with Eq. (1), we obtain

$$\zeta_{-n} = \zeta_n = \begin{cases} 0 & \text{if } n \neq 1, \\ \dfrac{d}{2} & \text{if } n = 1. \end{cases}$$

Using these values in Eqs. (24)–(31), we can obtain the values of $H_{11}^{+}/H_0, H_{21}^{+}/H_0, H_{31}^{+}/H_0$, H_{41}^{+}/H_0; $H_{11}^{-}/H_0, H_{21}^{-}/H_0, H_{31}^{-}/H_0$and H_{41}^{-}/H_0, which are ratios of amplitude constants of the vertical component of displacement for irregularly reflected and transmitted waves to that of incident wave as

$$\frac{H_{11}^{+}}{H_0} = \frac{\Delta_{H11}^{+}}{\Delta_1^{+}}, \quad \frac{H_{21}^{+}}{H_0} = \frac{\Delta_{H21}^{+}}{\Delta_1^{+}}, \quad \frac{H_{31}^{+}}{H_0} = \frac{\Delta_{H31}^{+}}{\Delta_1^{+}}, \quad \frac{H_{41}^{+}}{H_0} = \frac{\Delta_{H41}^{+}}{\Delta_1^{+}},$$

$$\frac{H_{11}^{-}}{H_0} = \frac{\Delta_{H11}^{-}}{\Delta_1^{-}}, \quad \frac{H_{21}^{-}}{H_0} = \frac{\Delta_{H21}^{-}}{\Delta_1^{-}}, \quad \frac{H_{31}^{-}}{H_0} = \frac{\Delta_{H31}^{-}}{\Delta_1^{-}}, \quad \frac{H_{41}^{-}}{H_0} = \frac{\Delta_{H41}^{-}}{\Delta_1^{-}}. \tag{35}$$

The ratios of the amplitude constants of the horizontal component of displacement for irregularly reflected and transmitted waves to that of incident wave are given by

$$\frac{G_{11}^{+}}{G_0} = -\frac{F_{11}^{+}}{F_1}\frac{\Delta_{H11}^{+}}{\Delta_1^{+}}, \quad \frac{G_{21}^{+}}{G_0} = -\frac{F_{21}^{+}}{F_1}\frac{\Delta_{H21}^{+}}{\Delta_1^{+}}, \quad \frac{G_{31}^{+}}{G_0} = \frac{F_{31}^{+}}{F_1}\frac{\Delta_{H31}^{+}}{\Delta_1^{+}}, \quad \frac{G_{41}^{+}}{G_0} = \frac{F_{41}^{+}}{F_1}\frac{\Delta_{H41}^{+}}{\Delta_1^{+}},$$

$$\frac{G_{11}^{-}}{G_0} = -\frac{F_{11}^{-}}{F_1}\frac{\Delta_{H11}^{-}}{\Delta_1^{-}}, \quad \frac{G_{21}^{-}}{G_0} = -\frac{F_{21}^{-}}{F_1}\frac{\Delta_{H21}^{-}}{\Delta_1^{-}}, \quad \frac{G_{31}^{-}}{G_0} = \frac{F_{31}^{-}}{F_1}\frac{\Delta_{H31}^{-}}{\Delta_1^{-}}, \quad \frac{G_{41}^{-}}{G_0} = \frac{F_{41}^{-}}{F_1}\frac{\Delta_{H41}^{-}}{\Delta_1^{-}}, \tag{36}$$

where

$$\Delta_1^{+} = \begin{vmatrix} 1 & 1 & -1 & -1 \\ F_{11}^{+} & F_{21}^{+} & F_{31}^{+} & F_{41}^{+} \\ a_{11}^{+} & a_{21}^{+} & a_{31}^{+} & a_{41}^{+} \\ b_{11}^{+} & b_{21}^{+} & b_{31}^{+} & b_{41}^{+} \end{vmatrix}, \quad \Delta_1^{-} = \begin{vmatrix} 1 & 1 & -1 & -1 \\ F_{11}^{-} & F_{21}^{-} & F_{31}^{-} & F_{41}^{-} \\ a_{11}^{-} & a_{21}^{-} & a_{31}^{-} & a_{41}^{-} \\ b_{11}^{-} & b_{21}^{-} & b_{31}^{-} & b_{41}^{-} \end{vmatrix}.$$

The expressions of quantities $F_{11}^{\pm}, F_{21}^{\pm}, a_{11}^{\pm}, a_{21}^{\pm}, b_{11}^{\pm}, b_{21}^{\pm}$, etc. can be obtained from the expressions of quantities $F_{1n}^{\pm}, F_{2n}^{\pm}, a_{1n}^{\pm}, a_{2n}^{\pm}, b_{1n}^{\pm}, b_{2n}^{\pm}$, etc. by putting $n = 1$, respectively, the expressions of $\Delta_{H11}^{+}, \Delta_{H21}^{+}, \Delta_{H31}^{+}$ and Δ_{H41}^{+} can be written by replacing the first, second, third and fourth columns, respectively, of the determinant in Δ_1^{+} by the column matrix $[a_{10}^{+}\ a_{20}^{+}\ a_{01}^{+}\ b_{01}^{+}]^{\mathrm{t}}$ and the expressions of $\Delta_{H11}^{-}, \Delta_{H21}^{-}, \Delta_{H31}^{-}$ and Δ_{H41}^{-} can be written by replacing the first, second, third and fourth columns, respectively, of the determinant in Δ_1^{-} by the column matrix $[a_{10}^{-}\ a_{20}^{-}\ a_{01}^{-}\ b_{01}^{-}]^{\mathrm{t}}$. The expressions of $a_{01}^{\pm}$ and $b_{01}^{\pm}$ can be obtained from $a_{0n}^{\pm}$ and $b_{0n}^{\pm}$, respectively, by putting $n = 1$.Thus, the reflection coefficients: $R_{\mathrm{pp}^{\pm}}^{1}$ (for irregularly reflected qP-waves at angles $\theta_1^{\pm}$), $R_{\mathrm{ps}^{\pm}}^{1}$ (for irregularly reflected qSV-waves at angles $\phi_1^{\pm}$) and the transmission coefficients: $T_{\mathrm{pp}^{\pm}}^{1}$ (for irregularly transmitted qP-waves at angles $\delta_1^{\pm}$), $T_{\mathrm{ps}^{\pm}}^{1}$ (for irregularly transmitted qSV-waves at angles $\gamma_1^{\pm}$) are given by

$$R_{\mathrm{pp}^{\pm}}^{1} = \sqrt{\frac{1+(F_{11}^{\pm})^2}{1+F_1^2}}\frac{\Delta_{H11}^{\pm}}{\Delta_1^{\pm}}, \quad R_{\mathrm{ps}^{\pm}}^{1} = \sqrt{\frac{1+(F_{21}^{\pm})^2}{1+F_1^2}}\frac{\Delta_{H21}^{\pm}}{\Delta_1^{\pm}},$$

$$T_{\mathrm{pp}^{\pm}}^{1} = \sqrt{\frac{1+(F_{31}^{\pm})^2}{1+F_1^2}}\frac{\Delta_{H31}^{\pm}}{\Delta_1^{\pm}}, \quad T_{\mathrm{ps}^{\pm}}^{1} = \sqrt{\frac{1+(F_{41}^{\pm})^2}{1+F_1^2}}\frac{\Delta_{H41}^{\pm}}{\Delta_1^{\pm}}. \tag{37}$$

It can be seen that these coefficients are functions of the angle of incidence, initial stress, incremental elastic coefficients, amplitude of the corrugation and frequency of the incident waves.

7. Particular cases

(a) To reduce the problem at a corrugated interface between a homogeneous isotropic elastic solid half-space and a pre-stressed elastic solid half-space, we substitute $B'_{11} = B'_{22} = \lambda' + 2\mu', B'_{12} = B'_{21} = B'_1 = \lambda', A'_1 = A'_2 = Q' = A'_{01} = \mu', A'_3 = \lambda' + \mu'$ and $S'_{11} = S'_{22} = 0$ into the expressions and equations in the half-space M'. With these substitutions, the quantities in the half-space M' reduce to $c'_1 = \sqrt{\lambda' + 2\mu'/\rho'}, c'_2 = \sqrt{\mu'/\rho'}, F_3 = -\tan\delta, F_4 = \cot\gamma, F^{\pm}_{31} = -\tan\delta^{\pm}_1$ and $F^{\pm}_{41} = \cot\gamma^{\pm}_1$. In this case, the reflection and transmission coefficients of the regularly reflected and transmitted waves are given by Eq. (34) with the following modified values of a_3, a_4, b_3 and b_4 given by $a_3 = (P_0 + S\tan\delta)\mu', a_4 = (P_0 - L\cot\gamma)\mu', b_3 = -P_0\lambda'\tan\delta - (\lambda' + 2\mu')S$ and $b_4 = P_0\lambda'\cot\gamma - (\lambda' + 2\mu')L$.

The reflection and transmission coefficients at the corrugated interface for the first-order approximation for the case of a periodic interface are given by Eq. (37) with the following modified values:

$$a^+_{20} = a^-_{20} = \iota\frac{d}{2}\left[F_1R + F_1R\frac{H_1}{H_0} + F_2Q\frac{H_2}{H_0} + S\tan\delta\frac{H_3}{H_0} - L\cot\gamma\frac{H_4}{H_0}\right], \quad a^+_{31} = \mu'(P_0 + p) + \mu'S^+_1\tan\delta^+_1,$$

$$a^+_{41} = \mu'(P_0 + p) - \mu'L^+_1\cot\gamma^+_1, \quad a^+_{01} = \frac{d}{2}\iota\left[B_{11}F_1pP_0 - B_1pR + A_{01}P_0R - A_1F_0R^2 + \{-B_{11}F_1pP_0 + B_1pR - A_{01}P_0R + A_1F_1R^2\}\frac{H_1}{H_0} + \{-B_{11}pF_2P_0 + B_1pQ - A_{01}P_0Q + A_1F_2Q^2\}\frac{H_2}{H_0} + \{(\lambda' + 2\mu')pP_0\tan\delta + \lambda'pS - \mu'P_0S - \mu'S^2\tan\delta\}\frac{H_3}{H_0} + \{-(\lambda' + 2\mu')pP_0\cot\gamma + \lambda'pL - \mu'LP_0 + \mu'L^2\cot\gamma\}\frac{H_4}{H_0}\right],$$

$$b^+_{31} = -\lambda'(P_0 + p)\tan\delta^+_1 - (\lambda' + 2\mu')S^+_1, \quad b^+_{41} = \lambda'(P_0 + p)\cot\gamma^+_1 - (\lambda' + 2\mu')L^+_1,$$

$$b^+_{01} = \iota\frac{d}{2}\left[A_2pP_0 - A_{01}F_1pR + B_1F_1RP_0 - B_{22}R^2 + \{pA_2P_0 - F_1pA_{01}R + B_1F_2RP_0 - B_{22}R^2\}\frac{H_1}{H_0} + \{A_2pP_0 - F_2pA_{01}Q + B_1F_2QP_0 - B_{22}Q^2\}\frac{H_2}{H_0} + \{-p\mu'P_0 - p\mu'S\tan\delta + \lambda'S\tan\delta + (\lambda' + 2\mu')S^2\}\frac{H_3}{H_0} + \{-\mu'pP_0 + p\mu'L\cot\gamma - \lambda'LP_0\cot\gamma + (\lambda' + 2\mu')L^2\}\frac{H_4}{H_0}\right], \quad a^-_{31} = \mu'(P_0 - p) + \mu'S^-_1\tan\delta^-_1,$$

$$a^-_{41} = \mu'(P_0 - p) - \mu'L^-_1\cot\gamma^-_1,$$

$$a^-_{01} = \iota\frac{d}{2}\left[-pB_{11}F_1P_0 + pB_1R + A_{01}RP_0 - A_1F_1R^2 + \{(pF_1B_{11} - A_{01}R)P_0 + A_1F_1R^2 - pB_1R\}\frac{H_1}{H_0} + \{(B_{11}pF_2 - A_{01}Q)P_0 - pB_1Q + F_2A_1Q^2\}\frac{H_2}{H_0} + \{(-p(\lambda' + 2\mu')\tan\delta - \mu'S)P_0 - \lambda'pS - \mu'S^2\tan\delta\}\frac{H_3}{H_0} + \{(p(\lambda' + 2\mu')\cot\gamma - \mu'L)P_0 - (\lambda' + 2\mu')pL + \mu'L^2\cot\gamma\}\frac{H_4}{H_0}\right], \quad b^-_{31} = -\lambda'(P_0 - p)\tan\delta^-_1 + (\lambda' + 2\mu')S^-_1,$$

$$b^-_{41} = (P_0 - p)\lambda'\cot\gamma^-_1 + (\lambda' + 2\mu')L^-_1, \quad b^-_{01} = \iota\frac{d}{2}\left[\{(-pA_2 + F_1B_1R)P_0 + F_1pA_{01}R - B_{22}R^2 + \{(-pA_2 - B_1F_1R)P_0 + A_{01}F_1pR - B_{22}R^2\}\frac{H_1}{H_0} + \{(-pA_2 + B_1F_2Q)P_0 + F_2A_{01}pQ - B_{22}Q^2\}\frac{H_2}{H_0} + \{(\mu'p + S\lambda'\tan\delta)P_0 + \mu'pS\tan\delta + (\lambda' + 2\mu')S^2\}\frac{H_3}{H_0} + \{(p\mu' - L\lambda'\cot\gamma)P_0 - p\mu'L\cot\gamma + (\lambda' + 2\mu')L^2\}\frac{H_4}{H_0}\right].$$

These reflection and transmission coefficients at the corrugated interface for the first-order approximation of a periodic interface are functions of the angle of incidence, elastic parameters, initial stress, amplitude of the corrugated interface and frequency of the incident wave. When the amplitude of the corrugated interface is set equal to zero, i.e., when $d = 0$ in the formulae given in Eq. (37), it can be seen that each one of the coefficients corresponding to irregularly reflected and transmitted waves disappears as all these coefficients are proportional to the amplitude of the corrugated interface. It is because of the quantities $a_{10}^{\pm}, a_{20}^{\pm}, a_{01}^{\pm}$ and $b_{01}^{\pm}$ are proportional to the amplitude of the corrugated interface.

(b) If the elastic half-space M' is absent and the corrugation of the interface is neglected, then the problem reduces to the problem of reflection of a qP-wave at the free plane boundary of a homogeneous pre-stressed elastic half-space. For this, we substitute the quantities with a prime equal to zero and the reflection coefficients at a plane free surface of a pre-stressed elastic half-space are given by Eqs. $(34)_1$ and $(34)_2$ with the following modified values:

$$\Delta = a_2b_1 + b_2a_1, \quad \Delta_{H1} = a_2b_1 - b_2a_1, \quad \Delta_{H2} = 2a_1b_1.$$

These reflection coefficients exactly match those obtained by Sidhu and Singh [21] for the corresponding problem.

8. Computational results and discussion

The reflection and transmission coefficients are computed numerically at a periodic interface $z = d\cos px$ by taking the following values of relevant parameters in the half-spaces: in the half-space M: $B_{11} = 6.5 \times 10^{11}\,\mathrm{N/m^2}, B_{22} = 1.8 \times 10^{10}\,\mathrm{N/m^2}, B_{21} = -5.9 \times 10^{10}\,\mathrm{N/m^2}, Q = 2.5 \times 10^{11}\,\mathrm{N/m^2}$ and $S_{11} = S_{22} = 4.0\times 10^{10}\,\mathrm{N/m^2}$; in the half-space M': $B'_{11} = 2.1 \times 10^{11}\,\mathrm{N/m^2}, B'_{22} = 1.6 \times 10^{10}\,\mathrm{N/m^2}, B'_{21} = -2.7 \times 10^{10}\,\mathrm{N/m^2}, Q' = 4.5 \times 10^{11}\,\mathrm{N/m^2}$ and $S'_{11} = S'_{22} = 2.0 \times 10^{10}\,\mathrm{N/m^2}$ and the values of corrugation parameter and frequency parameter, respectively, are taken as $pd = 1.22 \times 10^{-4}$ and $\omega/pc_1 = 250.0$ wherever not mentioned.

For a given angle of incidence θ_0, one requires angles of reflected and transmitted qP and qSV-waves, i.e., angles θ and ϕ for reflected qP and qSV-waves, respectively, and angles δ and γ for transmitted qP and qSV-waves, respectively. This can be obtained from Snell's law given by Eq. (9), in which the dimensionless apparent velocity $\overline{c}$ is given by $\overline{c} = c_a/\beta = c_{1,2}/p_2\beta$. With this, the equation corresponding to two roots namely, c_1^2 and c_2^2 given in Eq. (4), can be written as

$$\overline{c}^4 - (\overline{E_1} + \overline{E_2})\overline{c}^2 + \overline{E_1E_2} - \overline{A}_3^2p_0^2 = 0, \tag{38}$$

$$\overline{E}_1 = \frac{E_1}{B_{11}p_2^2}, \quad \overline{E}_2 = \frac{E_2}{B_{11}p_2^2}, \quad \overline{A_3} = \frac{A_3}{B_{11}}, \quad p_0 = \frac{p_3}{p_2}, \quad \beta = \sqrt{\frac{B_{11}}{\rho}}, \quad (p_2, p_3) = (\sin\theta_0, \cos\theta_0).$$

From Eq. (38), we see that there are two roots of $\overline{c}^2$ corresponding to the velocities of qP- and qSV-waves, for a given value of θ_0. And for a given value of $\overline{c}$, there are two positive roots of θ_0 corresponding to the angles of the reflected qP- and qSV-waves. Substituting these values of $\overline{E_1}, \overline{E}_2$ and $\overline{A}_3$ into Eq. (38), we obtain

$$g_0p_0^4 + g_2p_0^2 + g_4 = 0, \tag{39}$$

where

$$g_0 = \frac{A_1}{B_{11}}\frac{A_2}{B_{11}}, \quad g_2 = \frac{A_1}{B_{11}}\frac{B_{22}}{B_{11}} + \frac{A_2}{B_{11}} - \frac{A_3^2}{B_{11}^2} - \left(\frac{A_1}{B_{11}} + \frac{A_2}{B_{11}}\right)\overline{c}^2, \quad g_4 = \overline{c}^4 - \left(1 + \frac{B_{22}}{B_{11}}\right)\overline{c}^2 + \frac{B_{22}}{B_{11}}.$$

Transforming the above equation by using $q = \frac{1}{p_0} = \frac{p_2}{p_3}$ we obtain

$$g_4q^4 + g_2q^2 + g_0 = 0. \tag{40}$$

There are two positive roots of this equation. The larger positive root will correspond to a reflected qP-wave and the smaller positive root will correspond to a reflected qSV-wave. Let $q1_3$ be the larger positive root and $q1_4$ be the smaller positive root of this equation. Thus, the corresponding angles of the reflected qP and qSV-waves are given by

$$\theta = \tan^{-1}(q1_3) \quad \text{and} \quad \phi = \tan^{-1}(q1_4),$$

A similar equation can be set up for transmitted waves in medium M' and the corresponding angles of the transmitted qP and qSV-waves are obtained as

$$\delta = \tan^{-1}(q1'_3) \quad \text{and} \quad \gamma = \tan^{-1}(q1'_4).$$

Thus, we have obtained the directions of the regularly reflected and transmitted waves. The directions of the irregularly reflected and transmitted waves can be computed by using the Spectrum theorem given in Eq. (10).

Now, we are ready to compute the reflection and transmission coefficients of the reflected and transmitted qP and qSV-waves numerically from the formulae given in Eqs. (34) and (37). The results are shown graphically in Figs. 2–17. In Figs. 2–13, the variation of the modulus of the reflection and transmission coefficients of the reflected and transmitted waves are depicted with the angle of incidence θ_0 at different values of the initial stresses. In these figures, curve I corresponds to the case when the initial stresses are zero (isotropic elastic case), curve II corresponds to the case when the values of initial stresses are taken as $S_{11} = S_{22} = 2.0 \times 10^{10}\,\mathrm{N/m^2}$, $S'_{11} = S'_{22} = 1.0 \times 10^{10}\,\mathrm{N/m^2}$, while curve III correspond to the case when the values of the initial stress are taken as $S_{11} = S_{22} = 4.0 \times 10^{10}\,\mathrm{N/m^2}$, $S'_{11} = S'_{22} = 2.0 \times 10^{10}\,\mathrm{N/m^2}$ (only the mantissa parts of these numerical values are shown in the figure legends). In Fig. 2, the effect of initial stress on the reflection coefficient R_{pp} can be clearly noticed. We note that the coefficient R_{pp} increases with an increase of the initial stress at every angle of incidence except at grazing and normal incidences. At normal and grazing incidences, there is no effect of initial stress on R_{pp}. It can also be noted that for all the three cases considered, the reflection coefficient R_{pp} starts from a certain value at normal incidence and then it decreases with an increase of the angle of incidence θ_0 up to a certain value and afterwards, it increases with a further increase of the angle of incidence and approaches unity as θ_0 approaches 90°. Curves I–III have their minimum values, respectively, at 25°, 21° and 16° angles of incidence.

In Fig. 3, the reflection coefficient $R^1_{\mathrm{pp}^+}$ corresponding to the irregularly reflected qP-wave starts from a certain value at normal incidence and then, it decreases to the value zero at 18° angle of incidence for isotropic case as shown in curve I and to the minimum value at 27° and 58° angles of incidence for non-zero initial stress cases as shown in curves II and III, respectively. Thereafter, curves I and II are parabolic in nature in the range $18^\circ \leqslant \theta_0 \leqslant 85^\circ$ and $27^\circ \leqslant \theta_0 \leqslant 82^\circ$ attaining the maximum value at 61° and 59° angles of incidence, respectively, while curve III increases with an increase of the angle of incidence. In Fig. 4, the value of reflection coefficient $R^1_{\mathrm{pp}^-}$ corresponding to an irregularly reflected qP-wave is greater in the isotropic elastic case than those in the

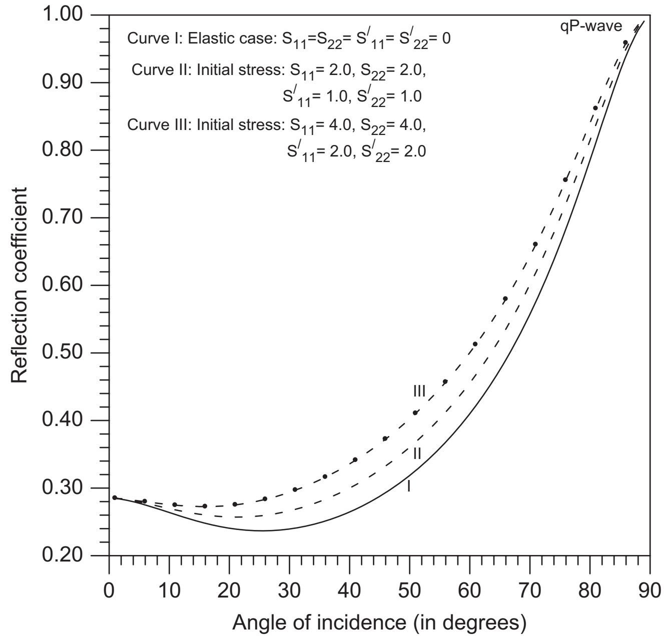

Fig. 2. Variation of reflection coefficient R_{pp} of a regularly reflected qP-wave with θ_0.

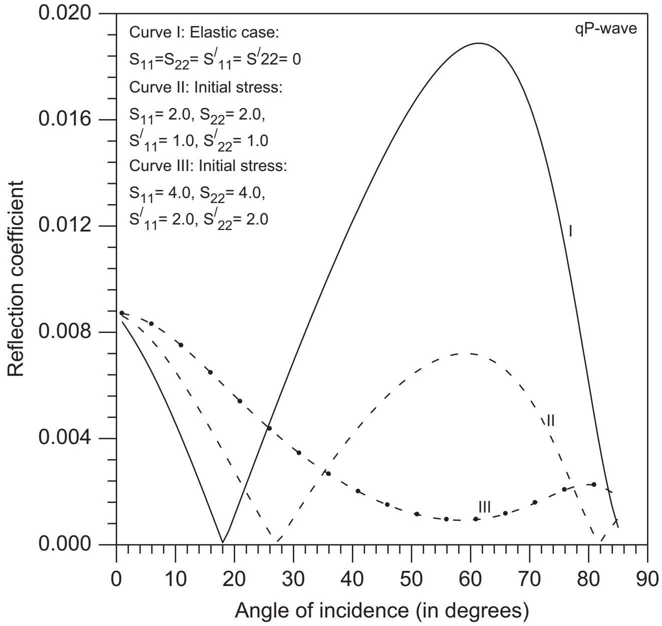

Fig. 3. Variation of reflection coefficient $R^1_{pp^+}$ of an irregularly reflected qP-wave with θ_0.

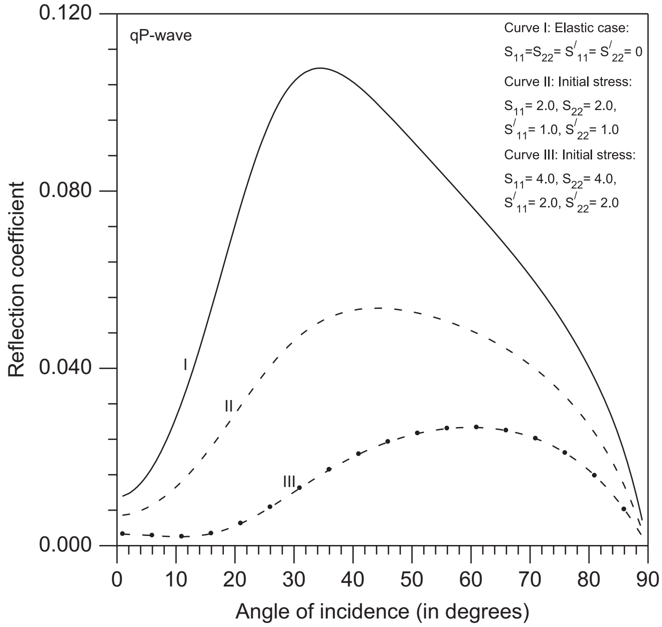

Fig. 4. Variation of reflection coefficient $R^1_{pp^-}$ of an irregularly reflected qP-wave with θ_0.

cases with initial stresses at each angle of incidence except at grazing incidence. However, it has been observed that with the increase of parameters corresponding to initial stresses, this reflection coefficient decreases. The effect of initial stress on the transmission coefficients T_{pp} and $T^1_{pp^+}$ corresponding to the regularly and irregularly transmitted qP-waves can be seen clearly from Figs. 5 and 6. The effect of initial stress is maximum at normal incidence, while it is negligible at grazing incidence. However, the coefficient T_{pp} increases, while the coefficient $T^1_{pp^+}$ decreases with an increase of initial stress. We observe from Fig. 7, that the behavior of

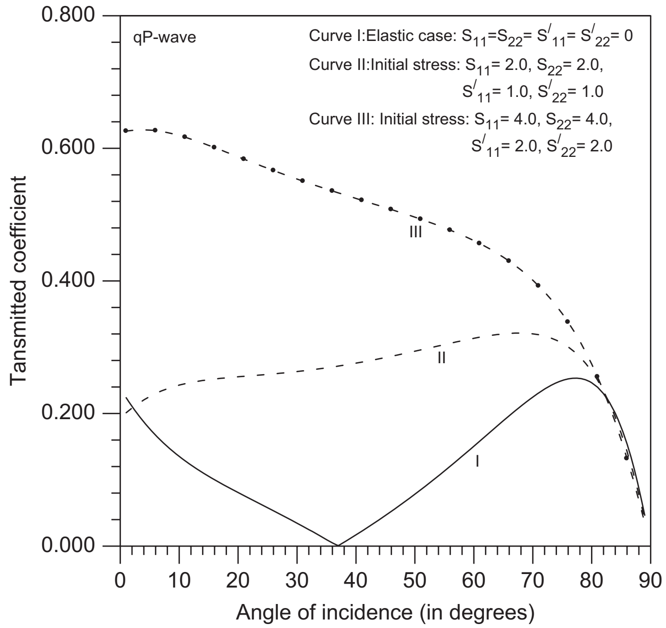

Fig. 5. Variation of transmission coefficient T_{pp} of a regularly transmitted qP-wave with θ_0.

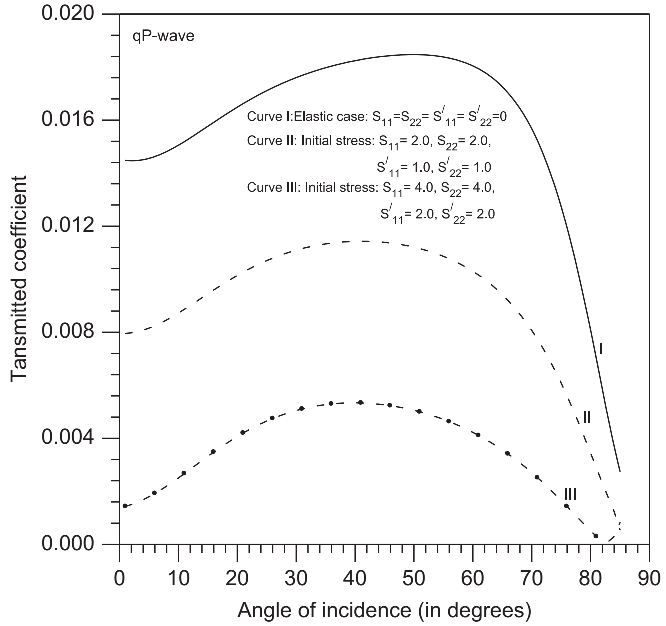

Fig. 6. Variation of transmission coefficient $T^1_{\text{pp}^+}$ of an irregularly transmitted qP-wave with θ_0.

transmission coefficient $T^1_{\text{pp}^-}$ corresponding to an irregularly transmitted qP-wave with the angle of incidence is similar to $R^1_{\text{pp}^-}$ as shown in Fig. 4. In Figs. 8, 10, 11 and 13, the modulus of the reflection coefficients R_{ps} and $R^1_{\text{ps}^-}$ corresponding to the reflected qSV-waves and the transmission coefficients T_{ps} and $T^1_{\text{ps}^-}$ corresponding to the transmitted qSV-waves increase from the value zero at normal incidence, with the increase of the angle of incidence attaining their maxima at a certain intermediate angle of incidence. Thereafter, they decrease with further increase of the angle of incidence, approaching the value zero as θ_0 approaches the vicinity of 90° angle

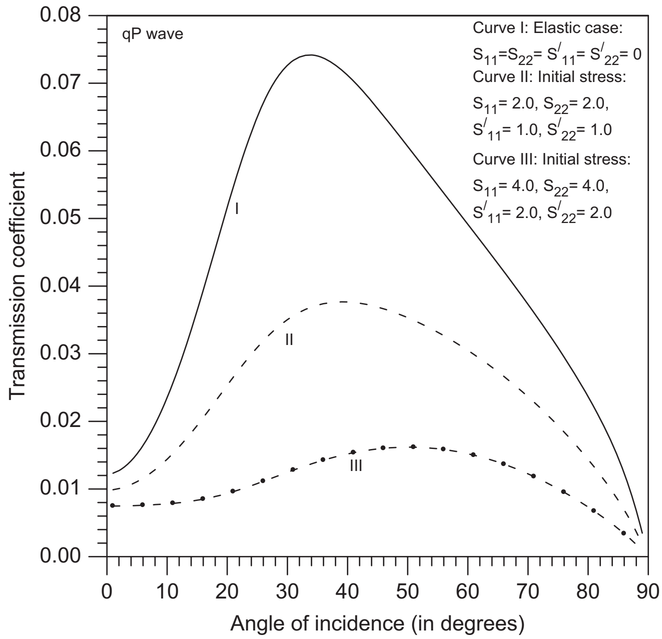

Fig. 7. Variation of transmitted coefficient $T^1_{pp^-}$ of an irregularly transmitted qP-wave with θ_0.

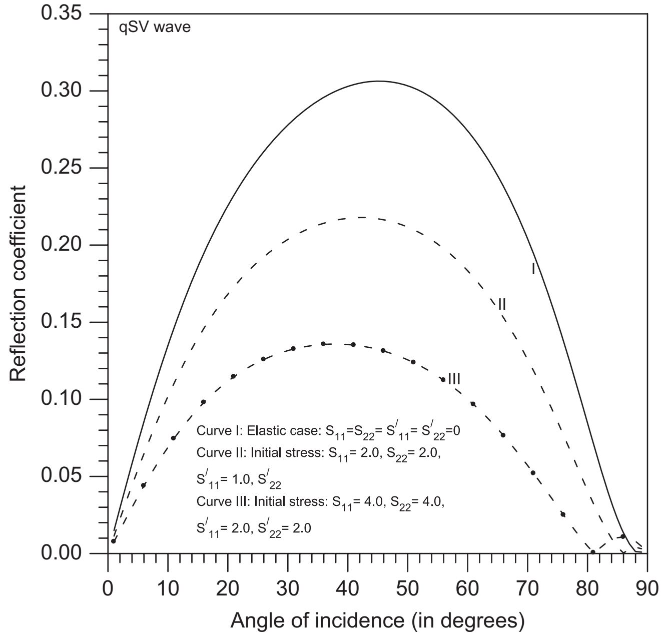

Fig. 8. Variation of reflection coefficient R_{ps} of a regularly reflected qSV-wave with θ_0.

of incidence. We note from Figs. 9 to 12 that the coefficients $R^1_{ps^+}$ and $T^1_{ps^+}$ corresponding to irregularly reflected and transmitted qSV waves have a similar pattern with the angle of incidence. These are also influenced by the initial stress in the same fashion.

Figs. 14–17 show the variation of the modulus of reflection and transmission coefficients of the irregularly reflected and transmitted qP and qSV-waves with corrugation parameter pd and frequency parameter ω/pc_1 when the qP-wave is made incident at 15° angle of incidence. In these figures, we note that the reflection and

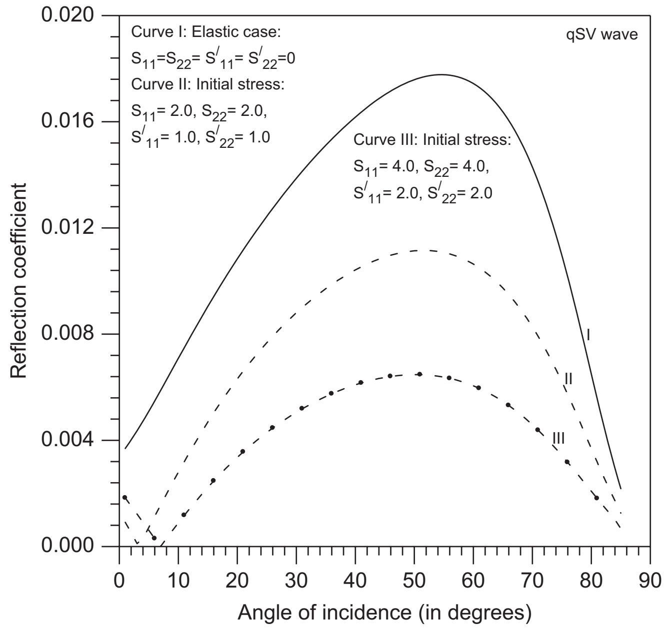

Fig. 9. Variation of reflection coefficient $R^1_{ps^+}$ of an irregularly reflected qSV-wave with θ_0.

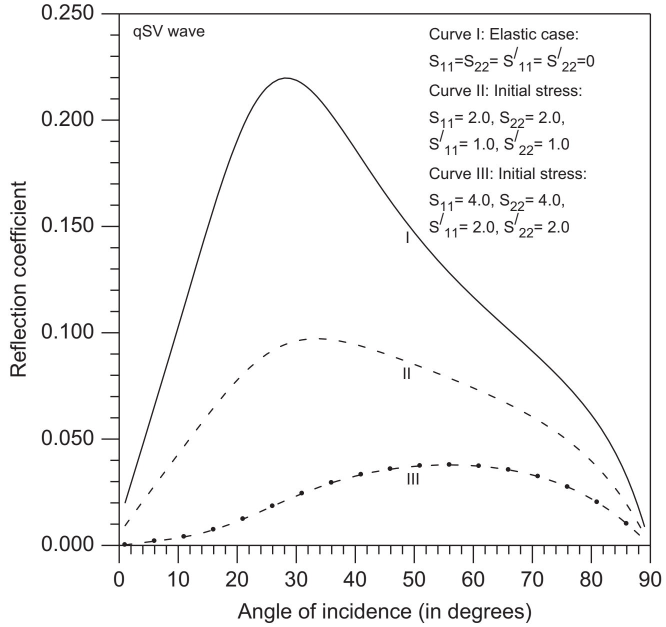

Fig. 10. Variation of reflection coefficient, $R^1_{ps^-}$ of an irregularly reflected qSV-wave with θ_0.

transmission coefficients corresponding to irregularly reflected and transmitted waves increase linearly with an increase of corrugation and frequency parameters. In Figs. 16 and 17, the linear increase of the reflection and transmission coefficients with the frequency parameter ω/pc_1 is because the reflection and transmission coefficients corresponding to the irregularly reflected and transmitted waves are proportional to the amplitude of the corrugation. These results are similar to the results obtained in Asano [3] and Gupta [5] in their problems.

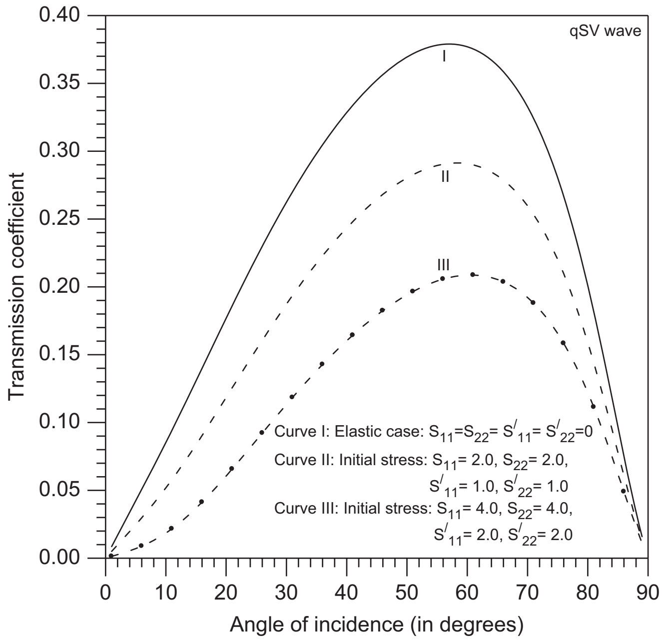

Fig. 11. Variation of transmission coefficient, T_{ps} of a regularly transmitted qSV-wave with θ_0.

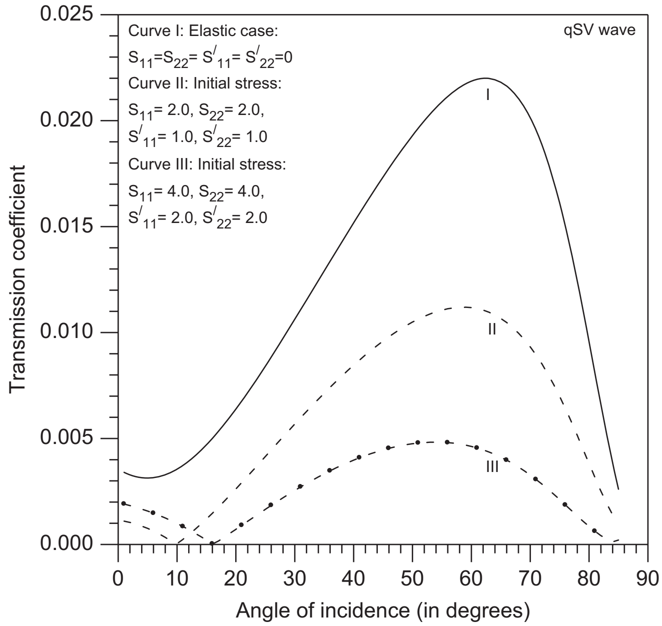

Fig. 12. Variation of transmission coefficient, $T^{1}_{\mathrm{ps}+}$ of an irregularly transmitted qSV-wave with θ_0.

9. Conclusion

The reflection and transmission coefficients due to a plane qP-wave incident at a corrugated interface between two pre-stressed elastic half-spaces are obtained. Rayleigh's method of approximation is adopted in order to find out these coefficients for first-order approximation of the corrugation. The solutions of

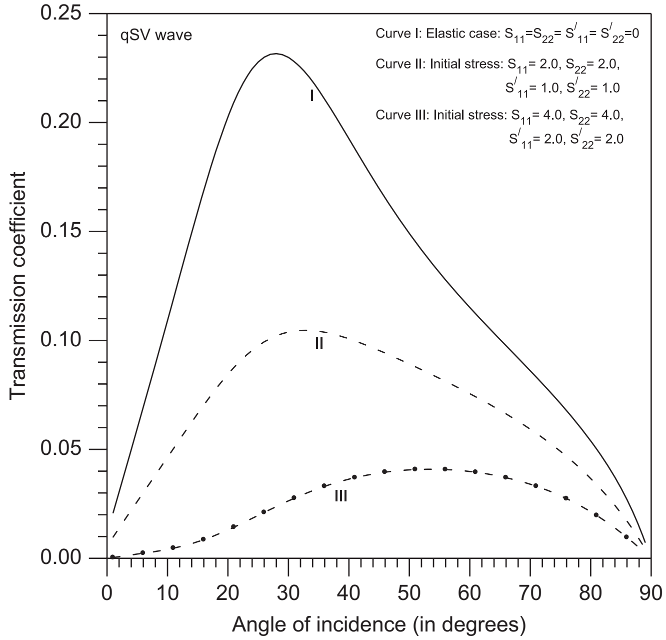

Fig. 13. Variation of transmission coefficient, $T^1_{ps^-}$ of an irregularly transmitted qSV-wave with θ_0.

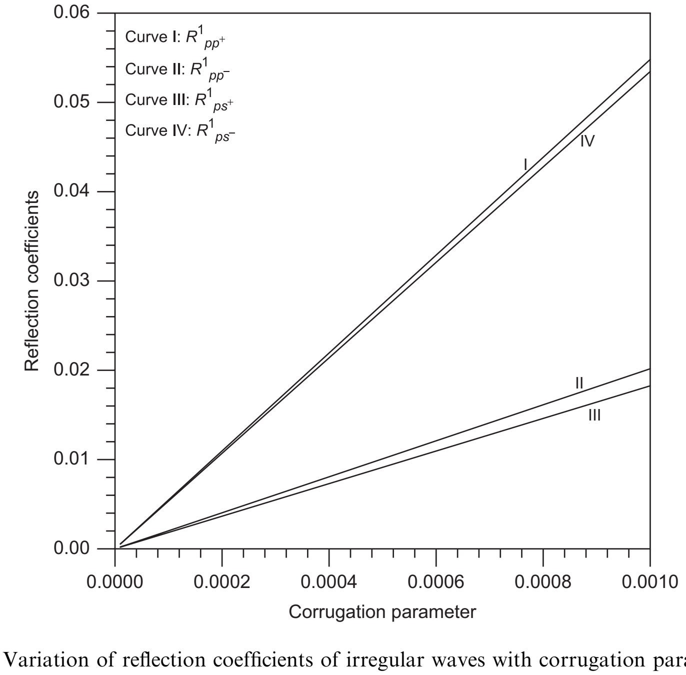

Fig. 14. Variation of reflection coefficients of irregular waves with corrugation parameter *pd*.

first-order approximation of these coefficients are expressed in closed form for a periodic type of interface (cosine law). It is concluded that

(i) the reflection and transmission coefficients are functions of the initial stresses, incremental elastic coefficients and angle of incidence,

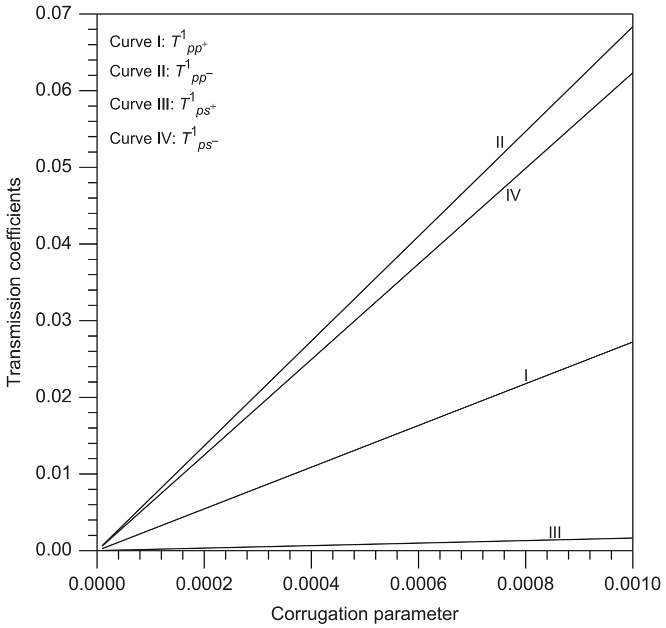

Fig. 15. Variation of transmission coefficients of irregular waves with corrugation parameter *pd*.

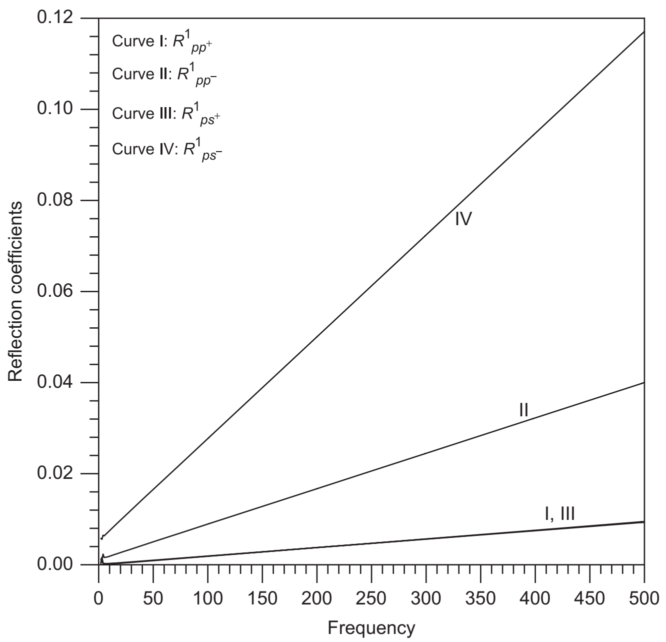

Fig. 16. Variation of reflection coefficients of irregular waves with frequency parameter ω/pc_1.

(ii) the reflection and transmission coefficients corresponding to the plane interface are independent of the corrugation and frequency of the incident waves,

(iii) the reflection and transmission coefficients of the irregularly reflected and transmitted qP- and qSV-waves are proportional to the amplitude of the corrugated interface,

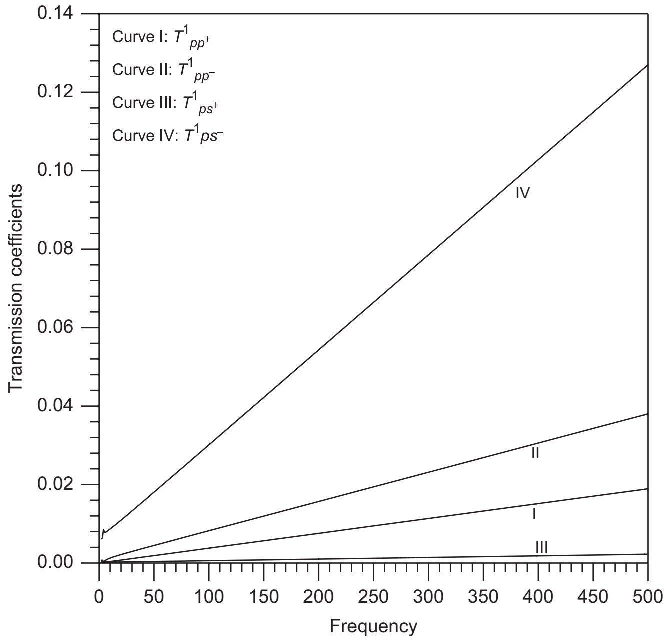

Fig. 17. Variation of transmission coefficients of irregular waves with frequency parameter ω/pc_1.

(iv) the coefficients corresponding to the regularly reflected and transmitted qP-waves are found to increase with an increase of initial stresses, while the coefficients of regularly reflected and transmitted qSV-waves are found to decrease with an increase of initial stresses,

(v) the reflection and transmission coefficients corresponding to the irregularly qP- and qSV- waves are found to decrease, in general, with an increase of initial stresses and

(vi) at grazing and normal incidences, no effect of initial stresses is observed on regularly reflected qP-, qSV-waves and regularly transmitted qSV-waves. However, a significant effect of initial stress is noticed on regularly transmitted qP-waves at normal incidence.

Acknowledgments

One of the authors (S.S. Singh) is thankful to University Grants Commission, New Delhi for providing financial support in the form of Research Fellowship under the Research Fellowship in Sciences for Meritorious Students (RFSMS) Scheme for completing this study.

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